Physics 3540 Final Exam (150 points)  
Modern Physics  
May 20, 2014  
Name ________________________  
Mail No. ________________________

**INSTRUCTIONS:** No references other than the constants and equations on the attached Final Equations/Constants Sheet. Points are indicated in parentheses. GOOD LUCK!

1. (15) If astronauts could travel at \( v = 0.95c \), we on Earth would say it takes \( \frac{4.2}{0.95} = 4.4 \) years to reach Alpha Centauri 4.2 light years away. The astronauts disagree.  
(a) (7) How much time passes on the astronauts' clocks?  
(b) (8) What distance to Alpha Centauri do the astronauts measure?

2. (20) A proton in a high-energy accelerator is given a kinetic energy of 50.0 GeV \((1 \text{ GeV} = \text{10}^9 \text{ eV})\). Determine  
(a) (6) \( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \)  
(b) (7) the speed of the proton and (c) (7) the momentum of the proton.

3. (15) (a) (5) What is the photoelectric effect? Be sure to draw a diagram for your discussion.  
(b) (10) A light source of wavelength \( \lambda \) illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.00 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.00 eV. What is the work function of the metal?

4. (20) (a) (6) Draw an energy level diagram for the hydrogen atom including lowest energies up through \( n = 4 \).  
(b) (6) Calculate the frequency of the photon emitted by a hydrogen atom making a transition from the \( n = 4 \) to the \( n = 3 \) state.  
(c) (8) Compare this result with the frequency of revolution for the electron in these two Bohr orbits.

5. (13) Show that the de Broglie wavelength of an electron accelerated from rest through a small potential difference \( V \) is given by \( \lambda = \frac{1.226}{\sqrt{V}} \) where \( \lambda \) is in nanometers and \( V \) is in volts.

6. (20) An electron of energy \( E \) is incident upon the finite barrier shown where \( E > U \). 
For the following, start with the *time-independent* Schrodinger equation.  
(a) (4) Solve for the wave-function to the left of the barrier.  
(b) (4) Solve for the wave-function within the barrier.  
(c) (3) Write down the wave-function to the right of the barrier.  
(d) (6) Write down and apply the boundary conditions to your wave-functions at \( x = 0 \) and at \( x = L \). Do not solve these equations; just write down the equations to be solved.  
(e) (2) Define the reflection coefficient \( R \) and the transmission coefficient \( T \).  
(f) (1) What relation must hold for \( R + T \)?  
(Note: This problem is the same as your Test 4 problem except for one major difference: \( E > U \); not \( E < U \)).

7. (12) A particle of mass \( m \) moves in a two-dimensional box of sides \( L \).  
(a) (6) Write expressions for the wave functions and energies as a function of the quantum numbers \( n_1 \) and \( n_2 \) (assuming the box is in the \( xy \) plane).  
(b) (6) Find the energies of the ground state and the first excited state. Is either of these states degenerate? Explain.

8. (35) In the time remaining, define and/or discuss to the best of your ability the following:  
(a) The Compton Effect (Be sure to draw a diagram.)  
(b) Frank-Hertz experiment.  
(c) Rutherford experiment  
(d) Quantum Operators (List and identify five operators.).  
(e) Correspondence Principle.  
(f) Quantum Simple Harmonic Oscillator potential energy function, wave functions, and energies.  
(g) The Hydrogen Atom potential energy function, wave functions, and energies.
Physics 1503
Final Exam
5/20/14

1. (a) $\vec{v} = \sqrt{v_x^2 + v_y^2}$
(b) $\vec{v} = \sqrt{(2.3^2 + 3.8^2)} = 4.27 \text{ m/s}$
(c) $E = \sqrt{m^2 v^2} = 1.1 \times 3 \times 10^6 \text{ eV}$

2. (a) $E_n = -1.36 \text{ eV}$
(b) $E_n = -0.35 \text{ eV}$
(c) $E_n = -1.5 \text{ eV}$
(d) $E_n = -3.4 \text{ eV}$

3. $N = \frac{1}{2} \pi r^2 h = \frac{1}{2} \pi \left(\frac{2.5 \times 10^{-5}}{2}ight)^2 \times 10^{-5} = 1.59 \times 10^{-8}$

4. (a) $T = \frac{1}{2} \pi r^2 h = \frac{1}{2} \pi \left(\frac{2.5 \times 10^{-5}}{2}ight)^2 \times 10^{-5} = 1.59 \times 10^{-8}$
(b) $T = \frac{1}{2} \pi r^2 h = \frac{1}{2} \pi \left(\frac{2.5 \times 10^{-5}}{2}ight)^2 \times 10^{-5} = 1.59 \times 10^{-8}$

5. (a) $E = \frac{1}{2} m v^2$
(b) $E = \frac{1}{2} m v^2$

6. (a) $\psi(x) = \psi(x) \psi(x)$
(b) $\psi(x) = \psi(x) \psi(x)$
(c) $\psi(x) = \psi(x) \psi(x)$

7. (a) $\psi(x) = \psi(x) \psi(x)$
(b) $\psi(x) = \psi(x) \psi(x)$
(c) $\psi(x) = \psi(x) \psi(x)$

8. (a) $\psi(x) = \psi(x) \psi(x)$
(b) $\psi(x) = \psi(x) \psi(x)$
(c) $\psi(x) = \psi(x) \psi(x)$

9. (a) $\psi(x) = \psi(x) \psi(x)$
(b) $\psi(x) = \psi(x) \psi(x)$
(c) $\psi(x) = \psi(x) \psi(x)$

10. (a) $\psi(x) = \psi(x) \psi(x)$
(b) $\psi(x) = \psi(x) \psi(x)$
(c) $\psi(x) = \psi(x) \psi(x)$

11. (a) $\psi(x) = \psi(x) \psi(x)$
(b) $\psi(x) = \psi(x) \psi(x)$
(c) $\psi(x) = \psi(x) \psi(x)$

Extra Space
a) Photon to the right will see a light packet striking an electron.

The compton effect refers to the light packet's loss of energy (lower freq) and the electron's gain in energy. The light packet experiences a \( \Delta \lambda = L \frac{h}{m_e c (1 - \cos \theta)} \). The following conservation equations hold:

\[
\begin{align*}
E_i - m_e c^2 &= E_{\text{electron}} + E_f \\
\text{Momentum} &= P_{\text{electron}} = P_{\text{photon}} + P_{\text{electron}}
\end{align*}
\]

\[
P_{\text{electron}} \sin \theta = P_{\text{photon}} \sin \theta
\]

b) The Frank-Hertz experiment showed the energy is quantized. Electrons were accelerated through a cloud of mercury. As the acceleration voltage increased, so did the current, until it suddenly fell! This happened cyclically (except for 0V). This corresponded to electrons possessing the right amount of energy to be absorbed by the Hg in a jump from a lower to higher energy state.

d) Quantum Operators correspond to the following sharp observables and are used with the wave \( \phi \): \( \langle \hat{A} \rangle = \langle \phi | \hat{A} | \phi \rangle \)

- Position \( \langle \hat{x} \rangle = [x] \)
- Momentum \( \langle \hat{p} \rangle = \frac{i}{\hbar} \frac{\partial}{\partial x} \)
- Kinetic Energy \( \langle \hat{K} \rangle = \frac{\hat{p}^2}{2m} \)
- Hamiltonian \( \langle \hat{H} \rangle = \frac{\hat{K}}{2m} + U \)
- Total Energy \( \langle \hat{E} \rangle = \frac{i}{\hbar} \frac{\partial}{\partial x} \)

Note \( \langle \hat{E} \rangle = \langle \hat{H} \rangle + \langle \hat{U} \rangle \) from Schrödinger's Equation.
5) c) The Rutherford Experiment confirmed the existence of an extremely dense, positively charged atomic nucleus. When high energy alpha particles were fired at a thin metal foil, they passed through only slightly perturbed. But the occasional alpha particle was reflected straight back! This could only be caused by an electromagnetic repulsion from a dense, positive nucleus.

C) Bohr's Correspondence Principle states that Quantum Mechanics should agree with Classical Physics on the macroscopic level i.e. \( \text{Im quantum} = \text{Classical} \). This is why electrons don't have observable wavelengths and such.

7) The potential energy function for quantum mechanics can be approximated by local potentials. Thus, \( U = \frac{1}{2} k x^2 + V(x) \) where
\[
\omega = \sqrt{\frac{k}{m}} \Rightarrow k = \omega^2 m; \quad U = \frac{1}{2} \omega^2 x^2 m
\]
Its energies are limited to \( E_n = (n + \frac{1}{2}) \hbar \omega \), \( n = 0, 1, 2, ... \)
Its wave function is given by the Gaussian
\[
\psi_n(x) = e^{-\alpha x^2} \quad \text{where} \quad \alpha = \frac{\hbar}{2m}
\]
The hydrogen atom potential energy function is \( U(r) = -\frac{k e^2}{r} \). This is a spherically symmetric function. Hence it is appropriate to use spherical coordinates.

Use the 3D form of Schrödinger time-independent Eqn

\[
-\frac{\hbar^2}{2m} \nabla^2 \psi(r, \theta, \phi) + U(r) \psi(r, \theta, \phi) = E \psi(r, \theta, \phi)
\]

Assume the usual product solution \( \psi(r, \theta, \phi) = R(r) \Theta(\theta) \Phi(\phi) \)

to get hydrogen atom wave functions:

\[
\psi(r, \theta, \phi, t) = R(r) Y_l^m(\theta, \phi) e^{-i\omega t}
\]

The "spherical harmonics" which include the associated Laguerre polynomials and the usual oscillation in time

\[
\Rightarrow \quad \text{Note that quantized integer values of } n, \ell, \text{ and } m \text{ will each give a different wave function.} \quad (\text{These 3 quantum } \ell \text{'s are then combined with } m_S = \pm \frac{1}{2})
\]

And finally, the energy (miraculously) ends up only upon \( n \) — namely,

\[
E_n = -\frac{13.6 \text{eV}}{n^2}
\]
Final Equations/Constant Sheet

CONSTANTS:  
\( e = 1.602 \times 10^{-19} \text{ C} \)  
\( m_{\text{electron}} = 9.107 \times 10^{-31} \text{ kg} \)  
\( c = 3.00 \times 10^8 \text{ m/s} \)  
\( 1.000 \ u = 1.66054 \times 10^{-27} = 931.494 \text{ MeV/c}^2 \)  
\( 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \)  
\( h = 6.626 \times 10^{-34} \text{ J s} = 4.136 \times 10^{-15} \text{ eV s} \)

\[ \text{EQUATIONS:} \]

\( f_{\text{observed}} = \frac{1}{1 + \frac{v}{c}} f_{\text{source}} \)

\[ x' = \gamma (x - vt) \]

\[ t' = \frac{\gamma (t - \frac{vx}{c^2})}{c^2} \]

\[ u'_y = \frac{u_y}{\gamma (1 - \frac{u_x}{c^2})} \]

\[ u'_z = \frac{u_z}{\gamma (1 - \frac{u_x}{c^2})} \]

\[ \lambda' - \lambda = \frac{h}{m_{e}c} (1 - \cos \theta) \]

\[ d \sin \phi = n \lambda \]

\[ \frac{e}{m} = \frac{V \theta}{1B^2d} \]

\[ a = \sqrt{\frac{9 \pi \nu}{2 \rho_{o} g}} \]

\[ q = \frac{mg}{E} \left[ \frac{v + v'}{v} \right] \]

\[ a_0 = \frac{h^2}{m_{ke}^2} = 0.5292 \times 10^{-10} \text{ m} \]

\[ r_n = \frac{n^2}{Z} a_0 \]

\[ \frac{1}{\lambda} = \frac{1}{\frac{1}{n_f^2} - \frac{1}{n_i^2}} \]

\[ E_n = \frac{(-13.6 \text{eV}) Z^2}{n^2} \]

\[ \psi(x, y, z) = \frac{8}{L_1 L_2 L_3} \sin \left( \frac{n_1 \pi x}{L_1} \right) \sin \left( \frac{n_2 \pi y}{L_2} \right) \sin \left( \frac{n_3 \pi z}{L_3} \right) \]

\[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + U(x) \psi = i\hbar \frac{\partial \psi}{\partial t} \]