1. (18) Choose any one of the five potential energy functions that we have studied. [Hint: The potential energy functions that we have studied so far include the infinite square well, the harmonic oscillator, the free particle, the delta function potential, and the finite square well.]
   (a) Write down and provide a graph of the potential energy function that you choose and clearly label the axes.
   (b) Write down the corresponding "form" of the solution for $\psi(x)$ of the time-independent Schrödinger Equation and also the general solution $\Psi(x,t)$. You need not normalize these functions.
   (c) Write down the energy for the nth stationary state (if bound) or the energy of the scattering state (if not bound).

2. (5) Evaluate: $\int_{-2}^{2} (x^2 + 2x - 3)\delta(x + 5)\,dx$ and $\int_{-5}^{5} (x^2 + 2x - 3)\delta(x - 3)\,dx$

3. (10) Find the eigenvalues and eigenvectors of the following matrix: $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

4. (30) (a) Write down in operator form the time-independent Schrödinger Equation starting with $\hat{H}\psi = E\psi$. Write the operator $\hat{H}$ in 3 dimensions in spherical coordinates. (b) Now use separation of variables to break the equation into two different equations: one a radial part and the other an angular part. (c) What are the solutions to the angular part called? (d) What conditions must be placed on $l$ and $m$? (e) Write down the potential energy function for the hydrogen atom. (f) What then are the solutions for the radial part called?

5. (5) Designate with up and down arrows (designating spin up and spin down respectively) two spin ½ particles and the $S = 0$ singlet configuration and do the same thing for two spin ½ particles and the $S = 1$ triplet configuration. Which of these is antisymmetric?

6. (6) From the periodic chart, write down the electronic orbital configuration of the element Zr (atomic number 40). (Recall: $s = 6$, $p = 1$, $d = 2$, $f = 3$, $g = 4$).

7. (5) Suppose that you have two particles with spins: $S_1 = 2$ and $S_2 = 5/2$. What spins are available in their combined eigenvectors?

8. (32) As usual, define and discuss the following terms:
   a. Wave function $\Psi(r,t)$.
   b. Phase velocity as compared to group velocity and the "dispersion" relation.
   c. The Fourier transform of $f(x)$ and the inverse Fourier transform of $F(k)$. (Plancherel's Theorem)
   d. Wave Packet and what is special about a "Gaussian" wave packet
   e. The Schwarz inequality as applied to vectors and why it is useful.
   f. The eigenfunctions and eigenvalues of $L^2$ and $L_z$; what values of $m$ are allowed; and what classical analogue might be used to describe $L^2$ and $L_z$.
   g. Larmor precession and the Larmor frequency.
   h. The Stern-Gerlach experiment and its significance. Be sure to include to significance of the inhomogeneous magnetic field.
9. (3 points each multiple-choice question or $13 \times 3 = 39$ points) Please provide the “best” answer for each of the following questions taken from the GRE or the ETS quantum physics sections:

1. De Broglie hypothesize that the linear momentum and wavelength of a free massive particle are related by which of the following constants?
   (a) Planck’s constant
   (b) Boltzmann’s constant
   (c) The Rydberg constant
   (d) The speed of light
   (e) Avogadro’s number

2. An atom has filled $n = 1$ and $n = 2$ levels. How many electrons does the atom have?
   (a) 2
   (b) 4
   (c) 6
   (d) 8
   (e) 10

3. A quantum mechanical harmonic oscillator has an angular frequency $\omega$. The Schrödinger equation predicts that the ground state energy of the oscillator will be
   (a) $-\frac{1}{2} \hbar \omega$
   (b) 0
   (c) $\frac{1}{2} \hbar \omega$
   (d) $\hbar \omega$
   (e) $3/2 \hbar \omega$

4. In the Bohr model of the hydrogen atom, the linear momentum of the electron at radius $r_n$ is given by which of the following? ($n$ is the principal quantum number.)
   (a) $n \hbar$
   (b) $nr_n \hbar$
   (c) $n \hbar / r_n$
   (d) $n^2 r_n \hbar$
   (e) $n^2 \hbar / r_n$

5. The normalized ground state wave function of hydrogen is
   $$\psi_{100} = \frac{2}{(4\pi)^{1/2}a_0^{3/2}} e^{-r/a_0}$$ where $a_0$ is the Bohr radius. What is the most likely distance that the electron is from the nucleus?
   (a) 0
   (b) $a_0/2$
   (c) $a_0/\sqrt{2}$
   (d) $a_0$
   (e) $2a_0$
6. The lifetime for the $2p \rightarrow 1s$ transition in hydrogen is $1.6 \times 10^{-9}$ s. The natural line width for the radiation emitted during the transition is approximately

(a) 100 Hz
(b) 100 kHz
(c) 100 MHz
(d) 100 GHz
(e) 100 THz

7. Which of the following statements about bosons and/or fermions is true?
   (a) Bosons have symmetric wave functions and obey the Pauli exclusion principle.
   (b) Bosons have antisymmetric wave functions and do not obey the Pauli exclusion principle.
   (c) Fermions have symmetric wave functions and obey the Pauli exclusion principle.
   (d) Fermions have antisymmetric wave functions and obey the Pauli exclusion principle.
   (e) Bosons and fermions obey the Pauli exclusion principle.

8. A particle is in an infinite square well potential with walls at $x = 0$ and $x = L$. If a particle is in the state $\psi(x) = A \sin \left( \frac{3\pi x}{L} \right)$, where $A$ is a constant, what is the probability that the particle is between $x = \frac{1}{3}L$ and $x = \frac{2}{3}L$?

(a) 0
(b) $\frac{1}{3}$
(c) $\frac{1}{\sqrt{3}}$
(d) $\frac{2}{3}$
(e) 1

Note from Schaum: $\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$

9. Which of the following are the eigenvalues of the Hermitian matrix $\begin{bmatrix} 2 & i \\ -i & 2 \end{bmatrix}$

(a) 1,0
(b) 1,3
(c) 2,2
(d) 1,-1
(e) 1+i,1-i

10. $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Consider the Pauli spin matrices $\sigma_x$, $\sigma_y$, and $\sigma_z$ and the identity matrix $I$ given above. The commutator $[\sigma_x, \sigma_y]$ is equal to which of the following?

(a) $I$
(b) $2i \sigma_x$
(c) $2i \sigma_y$
(d) $2i \sigma_z$
(e) 0
11. A spin $\frac{1}{2}$ particle is in a state described by the spinor $\chi = A^{\frac{1}{2}+i}$ where $A$ is a normalization constant. The probability of finding the particle with spin projection $S_z = -\frac{1}{2} \hbar$ is

(a) $\frac{1}{6}$  
(b) $\frac{1}{3}$  
(c) $\frac{1}{2}$  
(d) $\frac{2}{3}$  
(e) 1

12. Let $\hat{J}$ be a quantum mechanical angular momentum operator. The commutator $[\hat{J}_x \hat{J}_y, \hat{J}_z]$ is equivalent to which of the following?

(a) 0  
(b) $i\hbar \hat{J}_z$  
(c) $i\hbar \hat{J}_x \hat{J}_y$  
(d) $-i\hbar \hat{J}_x \hat{J}_z$  
(e) $i\hbar \hat{J}_x \hat{J}_y$

13. An electron with total energy $E$ in the region $x < 0$ is moving in the $+x$-direction. It encounters a step potential at $x = 0$. The wave function for $x \leq 0$ is given by

$$\psi = Ae^{i k_1 x} + Be^{-i k_1 x}, \text{ where } k_1 = \frac{2mE}{\hbar^2};$$

and the wave function for $x > 0$ is given by

$$\psi = Ce^{i k_2 x}, \text{ where } k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar}.$$ 

Which of the following gives the reflection coefficient for the system?

(A) $R = 0$  
(B) $R = 1$  
(C) $R = \frac{k_2}{k_1}$  
(D) $R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$  
(E) $R = \frac{4k_1k_2}{(k_1 + k_2)^2}$

THAT'S IT! HAVE AN EXCELLENT HOLIDAY BREAK, EVERYONE!  
Jerry
**Fundamental Equations**

Schrödinger equation:
\[ i\hbar \frac{\partial \psi}{\partial t} = H \psi \]

Time-independent Schrödinger equation:
\[ H \psi = E \psi, \quad \psi = \psi e^{-iEt/\hbar} \]

Hamiltonian operator:
\[ H = -\frac{\hbar^2}{2m} \nabla^2 + V \]

Momentum operator:
\[ p = -i\hbar \nabla \]

Time dependence of an expectation value:
\[ \frac{d\langle Q \rangle}{dt} = \frac{i}{\hbar} \langle [H, Q] \rangle + \left[ \frac{\partial \langle Q \rangle}{\partial t} \right] \]

Generalized uncertainty principle:
\[ \sigma_A \sigma_B \geq \left| \frac{1}{2i} \langle [A, B] \rangle \right| \]

Heisenberg uncertainty principle:
\[ \sigma_x \sigma_p \geq \frac{\hbar}{2} \]

Canonical commutator:
\[ [x, p] = i\hbar \]

Angular momentum:
\[ [L_x, L_y] = i\hbar L_z, \quad [L_y, L_z] = i\hbar L_x, \quad [L_z, L_x] = i\hbar L_y \]

Pauli matrices:
\[ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

**Mathematical Formulas**

Trigonometry:
\[ \sin(a \pm b) = \sin a \cos b \pm \cos a \sin b \]
\[ \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b \]

Law of cosines:
\[ c^2 = a^2 + b^2 - 2ab \cos \theta \]

Integrals:
\[ \int x \sin(ax) \, dx = \frac{1}{a^2} \sin(ax) - \frac{x}{a} \cos(ax) \]
\[ \int x \cos(ax) \, dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax) \]

Exponential integrals:
\[ \int_0^\infty e^{-x^2} \, dx = \frac{\sqrt{\pi}}{2} \]

Gaussian integrals:
\[ \int_0^\infty e^{-x^2/a^2} \, dx = \sqrt{\pi} \frac{(2n)!}{n!} \left( \frac{a}{2} \right)^{2n} \]
\[ \int_0^\infty e^{-x^2/a^2} \, dx = \frac{n!}{2} \left( \frac{a}{2} \right)^{2n+2} \]

Integration by parts:
\[ \int_a^b \frac{df}{dx} \, g \, dx = \left. f \right|_a^b - \int_a^b f \frac{dg}{dx} \, dx \]