A. Review of Springs

Before stretching

- \( F = kx \)
  - Hooke's Law
- Force by hand on spring
  - Force by spring on hand
  - \( F = -kx \) Note minus sign!

But force by spring on hand

- Spring constant \( k \)
- Mass \( m \)
- FBDO of block

\[ F = -kx \]
\[ ma = -kx \]
\[ a = -\frac{k}{m} x \]

Let \( \omega^2 = \frac{k}{m} \)
\[ a = -\omega^2 x \]

B. Let think through this motion

- Choose \( \chi = 0 \) (equilibrium position)

Here \( \chi = 0 \)
Here \( \chi = -A \)
Here \( \chi = -A \)
Here \( \chi = 0 \)
Here \( \chi = 0 \)

Block rushes through equilibrium
At max velocity, then starts slowing down
Block comes momentarily to rest

Here \( V = \text{max} \)

Here \( V = 0 \)

Maximum displacement \( A \) called the "amplitude"
C. This is NOT constant acceleration! One CANNOT use the Big 4 Equations for constant acceleration.

D. How can one mathematically get the motion equations $x = x(t)$, $v = v(t)$, and $a = a(t)$?

SOLID $a = -w^2 x$. ("w" has yet to be interpreted)

$$\frac{d^2 x}{dt^2} = -w^2 x$$

What function(s) have we learned that if you take its derivative twice, you get a "$w^2"times that same function?"

E. The answer:

$x = A \sin (wt + \theta)$ or $x = A \cos (wt + \theta)$

Either solution works! The book (p.439 uses the "cosine") I'll use the sine function.

Let's check it:

$x = A \sin (wt + \theta)$

$\theta$ is called the "phase." (When $t = 0$, this is used to give "$x" when we start the clock.)

The maximum displacement is called the "amplitude."

The angular freq. $w = \sqrt{\frac{k}{m}}$ (see previous page) and has units $\text{rad/s}$.

The frequency $f$ is the number of oscillations (or cycles) per second. $1 \text{ cycle/s} = 1 \text{ Hz} = 1 \text{ Hertz}$. 

F. Let's explain the terms!

$x = A \sin (wt + \theta)$
C. Angular Frequency \( \omega \), Frequency \( f \), and Period \( T \)

1. \( \omega \) is in \( \frac{\text{rad}}{\text{s}} \) but there are \( 2\pi \) radians in 1 cycle (or oscillation)

2. \( f \) is in \( \frac{\text{cycles}}{\text{s}} \)

   Quick Example:
   
   If \( f = 5 \text{ Hz} \), find \( \omega \)

   \[
   \omega = (\frac{5 \text{cycles}}{\text{s}})(\frac{2\pi \text{ rad}}{\text{cycle}}) = 10\pi \text{ rad/s}
   \]

   Or briefly \( \omega = 2\pi f \)

3. \( T \) is the time required for one complete cycle \( \frac{\text{cycle}}{\text{s}} \)

   Note: \( T = \frac{1}{f} \)

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H. Visualization of \( x = A \sin (\omega t + \theta_0) \)

- \( x = A \sin \omega t \)
- When \( t = T \)
  \( \omega T = 2\pi \)
  \( T = \frac{2\pi}{\omega} \)
- \( v = A\omega \cos \omega t \)
- \( a = -A\omega^2 \sin \omega t \)

\( a = -\omega^2 x \)
I. KINETIC ENERGY $K$ AND POTENTIAL ENERGY $U$.

1. NEGLECTING FRICTION, THE TOTAL ENERGY $E = K + U$

   OR $E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$

   $E = 18$ A CONSTANT.

2. IF $x = 0$, $E = \frac{1}{2} m v_{\text{max}}^2$ AND $\sqrt{2E} = \frac{m v_{\text{max}}}{m}$

3. IF $v = 0$, $E = \frac{1}{2} kA^2$ AND $A = \sqrt{\frac{2E}{k}}$

J. SUPPOSE YOU HAVE AN OSCILLATING SPRING WHERE $M = 25$ g, $k = 4N/m$, AND AT $t = 0$, $x_0 = 10$ cm AND $v_0 = 40$ cm/s.

   USE $x = A \sin (\omega t + \theta_0)$ FORM.

   FIND (a) $\omega$ (b) $f$ (c) $T$ (d) $E$ (e) $A$ (f) $N_{\text{max}}$ (g) PHASE $\theta_0$

SOL’NS:

(a) FIND THE ANGULAR FREQUENCY, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4 N/m}{0.025 kg}} = 4.0 \text{ rad/s}$

(b) FIND THE FREQUENCY, $f = \frac{\omega}{2\pi} = \frac{4.0 \text{ rad/s}}{2\pi \text{ rad/cycle}} = 0.64 \text{ Hz}$

(c) $T = \frac{1}{f} = \frac{1}{0.64 \text{ cycles/s}} = 1.57 \text{ cycle/s}$

(d) FIND THE TOTAL ENERGY $E = \frac{1}{2} m v_0^2 + \frac{1}{2} kx_0^2 = \frac{1}{2} \left(0.025 \text{ kg}\right) \left(-4 \text{ m/s}\right)^2 + \frac{1}{2} \left(4 \text{ N/m}\right) \left(10 \text{ cm}\right)^2$

   OR $E = 0.0020 J + 0.0020 J \text{ or } E = 0.0040 J$

(e) FIND THE AMPLITUDE $A$, $A = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2(0.004 J)}{0.4 N/m}} = 0.14 \text{ m}$

(f) FIND $N_{\text{max}}$, $N_{\text{max}} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.004 J)}{0.025 \text{ kg}}} = 0.57 \text{ m/s}$

(g) FIND $\theta_0$, $x = A \sin (\omega t + \theta_0)$

   At $t = 0$, $x_0 = A \sin \theta_0$

   $\sin \theta_0 = \frac{x_0}{A}$ OR $\theta_0 = \sin^{-1} \frac{10 \text{ cm}}{0.14 \text{ m}} = 45^\circ$