A. Correspondance Principle

1. As we proceed from the small size of atoms described by quantum theory to the large size of everyday mechanics described by Newton's law, as \( n \to \infty \)

\[
\lim_{n \to \infty} \text{(quantum physics)} = \text{classical physics}
\]

2. We have already seen this for relativity as \( u \ll c \)

\[
p = \sqrt{\mu u} \Rightarrow p = \mu u \quad \text{(Classical Values)}
\]

\[
KE = E - mc^2 = \gamma mc^2 - mc^2 \Rightarrow \frac{1}{2} \mu u^2 \quad \text{for} \; u \ll c
\]

3. Now let's return to the energy emitted by the atom. Classically, the frequency of the emitted radiation should be equal to the frequency of rotation of the electron in its orbital motion:

\[
f = \omega = \frac{2\pi}{\lambda}
\]

So as \( n \to \infty \), \( f = f' \)

\[\omega = \frac{2\pi}{f'}\]

\[\text{Frequency given off by antenna} = \text{frequency of oscillation of a charged object in SHM?}
\]

\[\text{Frequency} = \text{frequency of rotation of charge in a circular orbit} \]

\[W = 2\pi f'\]

This is not the case for atoms in their low energy state since \( \Delta E = hf \) where \( \Delta E \) is the difference between two energy levels!
(b) But as \( n \) becomes larger, the radii become larger. And the correspondence principle tells us that \( w = w' \) for large radii.

\[ L = m_e v R \approx v = \frac{1}{m_e} \]

(c) Let's see what the correspondence principle predicts for large radii where \( w = w' \) from p. 14-3.\( ^\circ \)

Recall

\[ E = -\frac{1}{2} \frac{k e^2}{r} \quad \text{and} \quad L = m_e v r = v = \frac{1}{m_e} \]

*The Bohr Model*

\[ \Sigma F_i = m v^2 \]

\[ \frac{k e^2}{r} = -\frac{1}{2} \frac{m e^2}{m_e} \cdot \frac{L^2}{r} = \frac{1}{2} \frac{m e^2}{m_e} \cdot \frac{L^2}{r} \]

\[ E = -\frac{1}{2} \frac{m e^2}{m_e} \cdot \frac{L^2}{r} = -\frac{1}{2} \frac{m e^2}{m_e} \cdot \frac{L^2}{r} \]

\[ E = \frac{1}{2} \frac{m e^2}{m_e} \cdot \frac{L^2}{r} = -\frac{1}{2} \frac{m e^2}{m_e} \cdot \frac{L^2}{r} \]

\[ \therefore \quad \frac{\partial E}{\partial L} = \frac{-m e^2}{2} \cdot \frac{L^2}{r} \]

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Hence the change in energy \( de \) with change in angular momentum \( \partial L \) becomes:

\[ \frac{\partial E}{\partial L} = \frac{-m e^2}{2} \cdot \frac{L^2}{r} \]

And we can also obtain \( \frac{\partial E}{\partial L} \) in terms of \( w \).

From I \[ L^2 = m e k e^2 r \]

Also \[ L^2 = \left( m e k e^2 r \right) \cdot \left( m v r \right) \]

From II \[ \frac{k e^2}{r} = m v^2 \]

\[ \therefore \quad L^2 = m e k e^2 r \]

From \( v = w r \)

\[ \frac{k e^2}{r} = m r^2 w \]

\[ \therefore \quad L^2 = m e k e^2 r \]

Plugging IV into III

\[ \frac{\partial E}{\partial L} = \frac{m e k e^2}{m e k e^2 w} \]

\[ \therefore \quad \frac{\partial E}{\partial L} = \frac{w}{w} \]

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Which means that the change in angular momentum for large orbits is always \( \frac{w}{2\pi} \).

\[ \therefore \quad L = m_e v R = \frac{h}{2\pi} \]

For \( n = \) large integers, as assumed by Bohr p. 14-2.
B. The Franck-Heit Experiment

1. Recall from our earlier discussion, we noted that an electron in an atom could be excited to a higher energy level via absorption of a light photon having energy \( \Delta E = h \nu \) where \( \Delta E \) was the difference in energy from the ground state to one of the excited states.

2. Could any atom also become excited by bombardment with a fast moving electron? (YES!). The Franck-Heit experiment was to determine if this could occur.

3. (a) The experimental set up.

(b) As the accelerating voltage is increased to 1.5 V, the potential energy is \( U = q'V_{ab} = -eV_{ab} = 1.5 \text{ eV} \) drop which goes into the kinetic energy \( K = 1.5 \text{ eV} \). If there were no atoms in the tube, the electrons would just be able to overcome the reverse polarity of 1.5 V of the collector and flow as current to the 1.5 V battery.

(c) Now increase the accelerating voltage more turn up the heat (\( \approx 180^\circ \text{C} \)) and introduce some Hg atoms via evaporation.
(d) There will now be collisions between the electrons and the Hg atoms. Initially these will be elastic collisions with almost all of the energy returned to the electrons. If you chart

![Current vs. Accelerating Voltage Graph]

you will see the current starting off.

(e) Now increase the voltage even more! Now the electron beam collisions will pack sufficient energy for the mercury atom electrons to be excited to the 1st excited state. This takes energy away from the electrons in the beam and they will not be collected. Hence, one will see a dip in the current!

![Current vs. Accelerating Voltage Graph]

(f) As one continues to increase the accelerating voltage, one sees a second dip corresponding to two successive inelastic collisions by the electrons before being collected, then a third dip corresponding to three successive inelastic collisions, etc.

![Current vs. Accelerating Voltage Graph]

(g) Hence the first excited state of Hg has a separation of about 5eV above the ground state energy!
C. A final example: Suppose that an excited hydrogen atom de-excites from the \( n = 2 \) state to the \( n = 1 \) state.

1. Find the wavelength \( \lambda \), frequency \( f \), and energy \( E \) of the emitted photon.

   **Solution.** \( \lambda = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = R \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = R \frac{3}{4} \)

   \[ \lambda = \frac{4 \pi}{\lambda} = \frac{4 \left( \frac{1}{1.097 \times 10^{-8} \text{m}} \right)}{1.215 \times 10^{-7} \text{m}} = 121.6 \text{ nm} \]

   \[ c = \lambda f \quad \Rightarrow \quad f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{m/s}}{1.215 \times 10^{-7} \text{m}} = 2.47 \times 10^{15} \text{ Hz} \]

   \[ E = hf = (4.136 \times 10^{-15} \text{ eV/s})(2.47 \times 10^{15} \text{ Hz}) = 10.21 \text{ eV} \]

   Assuming all the energy goes to the photon.

   However, momentum must be conserved!

2. Find the momentum \( p \) and energy of the recoiling hydrogen atom.

   **Solution.** \( p_{\text{photon}} = \frac{E_{\text{photon}}}{c} \)

   \[ 10.21 \text{ eV} = \frac{1}{2} m_H \nu_H^2 + E_{\text{photon}} \]

   LARGE mass \( m_H \)

   Speed \( \nu_H \) will be very small!

   Let's assume that \( \frac{1}{2} m_H \nu_H^2 \ll E_{\text{photon}} \) \( + E_{\text{photon}} \approx 10.21 \text{ eV} \)

   Equating momenta after the photon is given off:

   \[ m_H \nu_H = \frac{E_{\text{photon}}}{c} \]

   or \( m_H \nu_H = \frac{10.21 \text{ eV}}{c} \)

   The momentum of the recoiling \( H \) is

   \[ K = \frac{1}{2} m_H \nu_H^2 = \frac{1}{2} \left( \frac{m_H \nu_H}{c} \right)^2 \]

   \[ K = 5.56 \times 10^{-8} \text{ eV} \]

   Answer: \( a = 1.62 \times 10^{-5} \text{ m} \) (partial) \( m = 1.92 \times 10^{-27} \text{ kg} \).

**END OF CHAPTER 4**